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# Salt Hydrates. IX. The Comparison of the Crystal Structure of Magnesium Sulfate Pentahydrate with Copper Sulfate Pentahydrate and Magnesium Chromate Pentahydrate

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The salt MgSO<sub>4</sub>. 5H<sub>2</sub>O crystallizes in space group PT, Z = 2, a = 6.314 (5), b = 10.505 (18), c = 6.030 (6) Å,  $\alpha = 81^{\circ}7 (12)', \beta = 109^{\circ}49 (12)', \gamma = 105^{\circ}5 (3)'$ . The crystal structure was refined using three-dimensional X-ray data (precession camera, Mo K $\alpha$  radiation) to R = 0.057 for 1366 observed structure factors. The Mg coordination octahedra and sulfate tetrahedra form chains of composition  $Mg(OH_2)_4SO_4$  extending parallel to [110]. The chains are connected to each other by hydrogen bonds, donated by the water oxygen atoms in the chain and by the fifth water molecule, which is not coordinated to an Mg atom. The salt should be formulated as [Mg(OH<sub>2</sub>)<sub>4</sub>SO<sub>4</sub>]. H<sub>2</sub>O. The hydrogen atom positions were determined from difference syntheses and were confirmed by a calculation of the positions of least electrostatic energy. The hydrogen atom positions in MgCrO<sub>4</sub>.  $5H_2O$  were calculated on the basis of geometrical arguments.  $MgSO_4.5H_2O$  is isostructural with  $CuSO_4.5H_2O$ , this being one of the few cases where a Cu salt and a Mg salt crystallize in the same structure type and in the same space group. The lengths of the hydrogen bonds in all three pentahydrates conform well with the extended electrostatic valence rule, which predicts that hydrogen bonds formed by water molecules bonded to a Cu ion in the square-planar coordination should be shorter than those donated by water molecules bonded to Mg ions. The comparison of the sulfate pentahydrates and of the difluorides of Mg and Cu suggests that the Jahn-Teller effect is not a sufficient condition for explaining the low lattice symmetries of Cu salts as compared with analogous Mg salts.

## Introduction

Magnesium sulfate can be crystallized out of aqueous solution as the mono-, five-fourth-, di-, tri-, tetra-, penta-, hexa- and hepta-hydrate (Hodenberg & Kühn, 1967). Only the mono-, the hexa- and the hepta-hydrate, however, are stable in contact with their solutions. Pentahydrate crystals are metastable in contact with an MgSO<sub>4</sub> solution at all temperatures, but they can be stored for a prolonged time when separated from their solution and shielded from the atmosphere. Magnesium sulfate pentahydrate has even been reported (Palache, Berman & Frondel, 1951) to occur as a mineral (pentahydrite). Magnesium and divalent copper only rarely form strictly isostructural pairs of compounds; one pair is MgSO<sub>4</sub>.  $5H_2O$  and CuSO<sub>4</sub>.  $5H_2O$ . By 'strictly isostructural', we mean in this context that two compounds have the same crystal structure and crystallize in the same space group. In many cases, Cu compounds analogous to an Mg compound do not exist (*e.g.* MgCO<sub>3</sub> is known, but CuCO<sub>3</sub> is not). When an analogous Cu compound does exist, usually either it has a completely different structure from that of the Mg compound, as in the case of CuO and MgO, or at least its crystal structure represents a distortion from the structure of the Mg compound, as in the case of  $CuF_2$  and  $MgF_2$  (see also Wells, 1962). The structures of  $CuF_2$  and  $MgF_2$  are not strictly isostructural because their symmetries are different even though the coordination numbers around the cations and anions are the same and the topology of the polyhedral connections is identical.

This is a continuation of our previous investigations of salt hydrates (VIII: Khan & Baur, 1972). The refinement part of this work represents a master's thesis of one of the authors (JLR) at the University of Pittsburgh, 1966.

#### Experimental

Crystals of MgSO<sub>4</sub>.5H<sub>2</sub>O were grown at 50°C from aqueous solution, which was acidified by the addition of H<sub>2</sub>SO<sub>4</sub>. The crystals decomposed rapidly after removal from the solution; consequently, they were coated with a thin layer of epoxy resin, which greatly reduced their decomposition rate. A crystal of approximate dimensions  $0.2 \times 0.4 \times 3.0$  mm, elongated parallel to the [001] direction and mounted with this zone axis parallel to the spindle axis, was used for data collection on a precession camera. Fourteen layers were recorded by the multiple-exposure technique in directions perpendicular to the c axis, [100], [010], [210], and [210]. About 2400 intensities  $[\sin \theta(\max)/\lambda = 0.85 \text{ Å}^{-1}]$  were estimated visually by comparison with a calibrated strip. The averaging and processing of the data resulted in 1625 unique  $F_{hkl}$  data, of which 206 were of zero intensity. The absorption correction was neglected  $(\mu(Mo K\alpha) = 5.7 \text{ cm}^{-1}, \mu R = 0.11 \text{ to } 0.22)$ . MgSO<sub>4</sub>. 5H<sub>2</sub>O crystallizes in space group  $P\overline{1}$ . The Mg atoms are in special positions 000 and  $\frac{1}{2}, \frac{1}{2}, 0$ . All other atoms are in general positions  $\pm (x, y, z)$ . Z = 2, F.W. 210.47,  $D_x = 1.93$  g.cm<sup>-3</sup>,  $D_m = 1.90$  g.cm<sup>-3</sup> (Hodenberg & Kühn, 1967). For unit-cell data, see Table 1. The cell parameters were derived from a careful measurement of calibrated precession photographs  $[\mu(M \circ K\alpha) =$ 0.7107 Å].

# Table 1. Unit-cell data for MgSO<sub>4</sub>.5H<sub>2</sub>O, MgCrO<sub>4</sub>. 5.H<sub>2</sub>O (Bertrand et al., 1971) and CuSO<sub>4</sub>.5H<sub>2</sub>O (Bacon & Curry, 1962)

The values measured for  $MgSO_4.5H_2O$  agree with the unitcell data reported by Hodenberg & Kühn (1967). Estimated standard deviations here and throughout the paper are in parentheses following each value.

	Mg, S salt	Mg, Cr salt	Cu, S salt
а	6·314 (5) Å	6·384 Å	6·141 Å
b	10.565 (18)	10.702	10.736
с	6.030 (6)	6.115	5.986
α	81°7 (12)'	81°33′	82°16′
β	109°49 (12)'	108°45′	107°26′
y Y	105°5 (3)'	104° <b>20</b> ′	102°40′
V	362·3 Å <sup>3</sup>	382·3 ų	366·4 ų

Starting with the positional parameters reported by Bacon & Curry (1962) for  $CuSO_4.5H_2O$ , the refine-

ment with isotropic temperature factors and unit weights proceeded to an R=0.123. At this point, a Hughes-type weighting scheme was introduced, the 14 interlayer scale factors were adjusted, and the data were re-averaged, using the improved scale factors. Refinement with anisotropic temperature factors vielded an R = 0.061. A three-dimensional difference synthesis revealed 10 maxima ranging in height from 0.33 to 0.67 e.Å<sup>-3</sup>, which were near the expected locations of the hydrogen atoms. The highest background peaks (near the position of the sulfur atom) were 0.4 $e.Å^{-3}$ . The hydrogen atoms were included in the leastsquares refinement, with constant isotropic temperature factors equivalent to the observed anisotropic temperature factors of the corresponding oxygen atoms. The R value dropped significantly to 0.057. The 53  $F_{obs}$  data that are believed to be affected by extinction were excluded from refinement. Observed and calculated structure factors are presented in Table 2, the heavy-atom parameters used for the calculation of the final structure factors are listed in Table 3 and the hydrogen-atom parameters are given in Table 4.

Computer programs employed in the course of the work are:

- LPFPRE: Lorentz-polarization factors for the precession camera, Baur, W.H., 1962, unpublished.
- ORFLS: crystallographic least-squares, (Busing, Martin & Levy, 1962) (in a local modification).
- FORDAP: Fourier synthesis (Zalkin, 1962, unpublished) (in a local modification).
- *ORFFE*: crystallographic function and errors, (Busing, Martin & Levy, 1964) (in a local modification).
- SADIAN 69: Bond distances, bond angles, and structure sketches, (Baur & Wenninger, 1969, unpublished).
- MANIOC: Electrostatic energies of crystals (Baur, 1965).
- CALHPO: Geometrical calculation of hydrogen atom positions (Baur, 1969, unpublished)

#### Discussion of the structure

The main features of the crystal structure of  $MgSO_4$ .  $5H_2O$  agree with those of  $CuSO_4$ .  $5H_2O$  as determined by Beevers & Lipson (1934) and confirmed by neutron diffraction by Bacon & Curry (1962). The two crystallographically different Mg atoms are octahedrally coordinated by four water oxygen atoms and two sulfate oxygen atoms each (Table 5). They share corners with the coordination tetrahedra around the S atoms to form chains of alternating octahedra and tetrahedra extending parallel to [110] (Fig. 1). These chains of composition Mg(OH<sub>2</sub>)<sub>4</sub>SO<sub>4</sub> contain four of the water molecules per formula unit, while the fifth one, O(W9), is not coordinated to a Mg atom. It is held in the structure by hydrogen bonds in an approxi-

mately tetrahedral arrangement. Of the ten hydrogen bonds formed by the five crystallographically independent water molecules, four reinforce the

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 $Mg(OH_2)_4SO_4$  chains within themselves, while the six remaining ones connect every chain with its six neighbor-chains (the arrangement of the chains around each other is approximately close-packed). The compound

can be formulated, in analogy to the Cu salt, as  $[Mg(OH_2)_4SO_4]$ . H<sub>2</sub>O.

Surprisingly, neither Beevers & Lipson (1934), nor Bacon & Curry (1962) have pointed out the chains of octahedra and tetrahedra in  $CuSO_4.5H_2O$ . They limited their attention to the immediate environments of the atoms. Consequently, this feature has not been

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# Table 2. MgSO<sub>4</sub>. 5H<sub>2</sub>O, observed and calculated structure factors (×10)

Values marked with an asterisk were given zero weight in the least-squares refinement.

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mentioned in books and review articles (see *e.g.* Wells, 1962, p. 587). By contrast, West (1934) pointed out the chains of face-shared octahedra of composition  $Li(OH_2)_3$  in  $Li(CIO_4).3H_2O$ , and these chains are mentioned in texts and review articles (see Wells, 1962, p. 583). This shows the importance of adequate descriptions of geometric and topologic features of crystal structures. Such descriptions are an aid in visualizing and classifying crystal structures.

The coordination octahedra around both Mg atoms can be formulated as  $[MgO_2(OH_2)_4]^{2+}$ , with the sulfate oxygen atoms in *trans* positions. Thus, they differ from the  $[MgO_2(OH_2)_4]^{2+}$  octahedra in MgSO\_4.4H<sub>2</sub>O (Baur, 1962) where the sulfate oxygen atoms were found in *cis* positions. The mean value of 2.063 Å for the bond distance Mg-O compares well with the corresponding distance of 2.065 Å in MgSO\_4.7H<sub>2</sub>O (Baur, 1964). Both octahedra are slightly, but significantly, distorted in such a way that the distance to the sulfate oxygen atoms is the longest. For the case of Mg(2)-O(2) [0.045 Å longer than Mg(2)-O(W7,W8)], this could be rationalized on the basis of the extended electrostatic valence rule (Baur, 1970), since O(2) is the oxygen atom receiving the highest bond strength of all oxygen atoms (see Table 6). This explanation is not possible for Mg(1)-O(1) [0.037 Å longer than Mg(1)-O(W5,W6)], because in the Mg(1) octahedron all oxygen atoms receive the same bond strength. Since the longer Mg bonds to O(1) and O(2) in MgSO<sub>4</sub>.5H<sub>2</sub>O correspond to the very long bonds, Cu-O in CuSO<sub>4</sub>. 5H<sub>2</sub>O, it is conceivable that this lengthening is caused by a structural control. The relationship between bond strength and bond length is better displayed by the sulfate group, where the distances [Table 5(a)] vary exactly as the bond strengths (Table 6). This is not surprising, because an analysis of the bond-strength bond-length correlations for 14 different coordinations (Baur, 1970) has shown that the correlation coefficient is the higher, the larger the formal valence of the central cation in question There fore, we should expect a better agreement for the  $S^{6+}$ -O bonds than for the Mg<sup>2+</sup>-O bonds.

The hydrogen bonds in MgSO<sub>4</sub>.5H<sub>2</sub>O are arranged as in CuSO<sub>4</sub>.5H<sub>2</sub>O [Table 5(c)]. The proposed hydrogen bonding scheme is substantiated by; (1) the analogy with CuSO<sub>4</sub>.5H<sub>2</sub>O, analyzed by neutron diffraction (Bacon & Curry, 1962); (2) the X-ray diffraction

Table 3. MgSO<sub>4</sub>. 5H<sub>2</sub>O, positional parameters in fractional coordinates and thermal parameters (both  $\times 10^4$ ) with their standard deviations

The definition of the Debye-Waller temperature factor is exp  $[-(\beta_{11}h^2 + \beta_{22}k^2 + \beta_{33}l^2 + 2\beta_{12}hk + 2\beta_{13}hl + 2\beta_{23}kl)]$ ; the equivalent isotropic temperature factor B is (Hamilton, 1959):

	x	У	Z	$\beta_{11}$	$\beta_{22}$	$\beta_{33}$	$\beta_{12}$	$\beta_{13}$	$\beta_{23}$	$B(A^2)$
Mg(1)	0000	0000	0000	93 (4)	18 (1)	78 (3)	-5(2)	-22 (3)	-2(1)	1.0 (1)
Mg(2)	5000	5000	0000	68 (3)	18 (1)	61 (3)	6 (2)	22 (2)	-2(1)	0.8 (1)
S	0371 (1)	2897 (1)	6624 (1)	60 (2)	17 (1)	53 (2)	3 (1)	12 (1)	0(1)	0·8 (1)
O(1)	9464 (4)	1509 (2)	7309 (3)	125 (6)	23 (2)	104 (5)	-10 (3)	14 (4)	15 (2)	1.5 (2)
O(2)	2680 (4)	3344 (2)	8331 (3)	83 (5)	28 (2)	128 (5)	-2(3)	- 19 (4)	-14 (2)	1.5 (2)
$\overline{O(3)}$	8808 (4)	3711 (2)	6610 (3)	126 (6)	46 (2)	128 (5)	44 (3)	36 (4)	-8(2)	1.6 (2)
<b>O</b> (4)	0590 (4)	2986 (2)	4256 (3)	135 (6)	38 (2)	74 (4)	-12(3)	- 56 (4)	-3 (2)	1.4 (2)
O(W5)	8279 (4)	0768 (3)	1639 (4)	192 (8)	39 (2)	186 (6)	-4 (4)	114 (5)	- 32 (3)	2.0 (2)
O(W6)	3086 (4)	1139 (3)	1848 (4)	149 (7)	32 (2)	119 (5)	-8(3)	12 (4)	-6(2)	1.8 (2)
O(W7)	4745 (4)	4042 (3)	3153 (4)	118 (6)	52 (3)	85 (5)	13 (3)	28 (4)	16 (2)	1.7 (2)
O(W8)	7567 (4)	4077 (3)	0318 (4)	116 (6)	63 (3)	104 (5)	47 (3)	22 (4)	-6(3)	1.7 (2)
O(W9)	4589 (4)	1324 (3)	6699 (4)	113 (7)	51 (3)	178 (6)	9 (3)	35 (5)	6 (3)	2.1 (2)

 $\frac{4}{3}(\beta_{11}a^2 + \beta_{22}b^2 + \beta_{33}c^2 + 2\beta_{12}ab\cos\gamma + 2\beta_{13}ac\cos\beta + 2\beta_{23}bc\cos\alpha).$ 

Table 4.  $MgSO_4.5H_2O$  and  $MgCrO_4.5H_2O$ , coordinates of the hydrogen atoms (×10<sup>3</sup>) as deduced from leastsquares refinement (LS), difference synthesis (Diff), calculation of least electrostatic energy (El), and geometrical calculations based on the assumed hydrogen bonding arrangement (Geom)

						MgS	O₄.5H2O						MgC	rO4 . 5	H <sub>2</sub> O
		LS			Diff El				Geom			Geom			
	x	у	Ζ	x	у	Z	x	у	z	x	У	z	x	у	z
H(54)	908 (15)	146 (8)	260 (13)	910	140	250	892	151	262	911	155	250	899	147	248
H(59)	724 (14)	023 (8)	230 (13)	730	020	220	703	017	212	737	016	253	733	010	247
H(62)	331 (13)	192 (7)	117 (12)	330	200	140	365	201	118	325	209	148	323	213	116
H(69)	336 (12)	148 (8)	321 (11)	330	140	350	377	110	355	354	095	355	353	112	335
H(73)	562 (13)	402 (8)	411 (11)	580	380	430	615	394	441	603	386	450	582	388	439
H(74)	350 (15)	392 (8)	349 (10)	360	380	380	344	375	376	331	368	352	316	367	331
H(83)	786 (13)	385 (9)	919 (14)	790	400	920	816	394	911	804	394	902	811	396	908
H(84)	842 (13)	374 (8)	179 (12)	850	400	160	862	389	187	852	373	179	854	377	179
H(91)	585 (13)	115 (8)	733 (11)	610	120	740	625	167	715	621	141	692	622	130	673
H(92)	433 (13)	201 (9)	706 (10)	420	200	720	385	196	698	440	206	734	458	206	721

evidence (see Table 4); (3) the calculation results of the positions of least electrostatic energy of the hydrogen atoms (Table 4) [the charges assumed on the different atoms were: magnesium, +2.0 e; sulfur, 0.0 e; sulfate oxygen, -0.5 e; water oxygen -1.0 e; hydrogen, +0.5 e. The shape of the water molecule was held constant (angle H–O(W)–H=109.5°, O(W)–H= 0.97 Å). The computation was performed in the same way as for CuSO<sub>4</sub>. 5H<sub>2</sub>O (Baur, 1965)]; (4) a consideration of the interatomic distances and angles between the O(W) atoms and the surrounding magnesium and oxygen atoms (see the following paper, Baur, 1972). The results of this geometrical calculation for

 $MgSO_4.5H_2O$  and  $MgCrO_4.5H_2O$  (Bertrand, Dusausoy, Protas & Watelle-Marion, 1971) are given in Table 4.

The distances and angles involving hydrogen atoms [Table 5(c)] are based, for MgSO<sub>4</sub>.5H<sub>2</sub>O, on the electrostatically calculated hydrogen atom positions, for MgCrO<sub>4</sub>.5H<sub>2</sub>O on the geometrical calculation, and for CuSO<sub>4</sub>.5H<sub>2</sub>O on the neutron diffraction results. The errors of the distances and angles involving hydrogen atoms for CuSO<sub>4</sub>.5H<sub>2</sub>O are  $\pm 0.01$  Å and  $\pm 1.0^{\circ}$  respectively. It is assumed that the positions calculated for the hydrogen atoms in MgSO<sub>4</sub>.5H<sub>2</sub>O are on the average within 0.1 Å of the positions as they could be determined by neutron diffraction, while for

 $MgCrO_4.5H_2O$  this estimated error margin is 0.2 Å (see Baur, 1972). The hydrogen bonding geometry

[Table 5(c)] is very similar for the three salts. Larger discrepancies occur for the  $H \cdots O(1)$  and  $H \cdots O(2)$ distances, in which O(W9) is the donor. These are the bonds which have the most pronounced differences in the  $O(W) \rightarrow O$  distances between the three compounds. They seem to be due to the different shape of the Cucoordination octahedron as compared to the Mg octahedra. The Cu(1)-O(1) and Cu(2)-O(2) distances are much longer than the Cu-O(W) distances; therefore, the tetrahedron is displaced in the Cu salt relative to its position in the Mg salts, in which the Mg octahedra are more regular. Similarly, the hydrogen bonds are affected by the larger size of the chromate group as compared with the sulfate group. Therefore, the position of the O(W9) atom and the lengths of its bonds are different in the three salts.

The mean length of all hydrogen bonds is 0.05 Å longer in the Mg salts than in the Cu salt [Table (5c)]. The shorter mean hydrogen bond length in the Cu salt is mainly due to the shortness of those  $O(W) \rightarrow O$  bonds which originate at the water molecules bonded into the square-planar coordination around the Cu ions. The same effect can be observed in the only other pair of isostructural hydrated Cu and Mg salts of which the crystal structures have been determined: Mg(NH<sub>4</sub>)<sub>2</sub>(SO<sub>4</sub>)<sub>2</sub>.6H<sub>2</sub>O (Margulis & Templeton, 1962) and Cu(NH<sub>4</sub>)<sub>2</sub>(SO<sub>4</sub>)<sub>2</sub>.6H<sub>2</sub>O (Brown & Chidambaram, 1969). In these salts, the hydrogen bonds donated by the water molecules bonded in the square-planar

# Table 5. Interatomic distances and bond angles in MgSO<sub>4</sub>. 5H<sub>2</sub>O, MgCrO<sub>4</sub>. H<sub>2</sub>O (Bertrand et al., 1971) and CuSO<sub>4</sub>. 5H<sub>2</sub>O (Bacon & Curry, 1962)

The e.s.d.'s involving nonhydrogen atoms are equal to or smaller than  $\pm 0.005$ ,  $\pm 0.02$  and  $\pm 0.02$  Å respectively for the bond lengths, and equal to or smaller than  $\pm 0.2$ ,  $\pm 0.7$  and  $\pm 1.0^{\circ}$  respectively for the bond angles.

For distances and angles involving hydrogen atoms see text.

(a)	Tetrahedral	group
(4)	renuncula	gioup

	Mg, S	Mg, Cr	Cu,S
	salt	salt	salt
T-O(1)	1·471 Å	1·64 Å	1.48
T-O(2)	1.477	1.65	1.49
T-O(3)	1.462	1.62	1.47
T-O(4)	1.467	1.64	1.50
Mean of 4	1.469	1.636	1.484
Range of 6 O–O	2.39-2.42	2.65-2.70	$2 \cdot 41 - 2 \cdot 44$
Mean of 6 O-O	2.399	2.671	2.423
Range of 6 O–T–O	108·2-110·8°	108·7-111·4°	108.3-111.0°
Mean of 6 O-T-O	109.5	109.5	109.5
(b) Octahedral groups			
$2 \times M(1) - O(W_6)$	2·049 Å	2.08 Å	1.98 Å
$2 \times M(1) - O(W5)$	2.059	2.03	1.99
$2 \times M(1) - O(1)$	2.091	2.08	2.38
$2 \times M(2) - O(W8)$	2.038	2.06	1.95
$2 \times M(2) - O(W7)$	2.046	2.04	1.98
$2 \times M(2) - O(2)$	2.097	2.08	2.43
Mean of 12	2.063	2.063	2.118
Mean of 8 short			
Cu-O(W) bonds			1.974
Mean of 4 long			
Cu–O bonds			2.407
Range of 24 O–O	2.844-3.002	2.88-2.97	2.77-3.19
Mean of 24 O–O	2.918	2.918	3.005
Range of 24 O-M-O	86·9–93·1°	88·6–91·4°	87·0-93·0°

(c) Water molecules			
	Mg, S	Mg, Cr	Cu, S
	salt	salt	salt
$O(W5) \rightarrow O(4)$	2·802 Å	2·78 Å	2·83 Å
$O(W5) \rightarrow O(W9)$	2.755	2.84	2.76
H(54) - O(4)	1.84	1.81	1.91
H(54) - O(W9)	1.83	1.89	1.81
O(4) - O(W5) - O(W9)	120·8°	122·0°	120·5°
$O(W_5) - H(54) - O(4)$	169	175	168
O(W5) - H(59) - O(W9)	160	166	171
$O(W_6) \rightarrow O(2)$	2.890 Å	2·91 Å	2∙80 Å
$O(W6) \rightarrow O(W9)$	2.773	2.81	2.75
H(62) - O(2)	2.07	2.07	1.90
H(69) - O(W9)	1.82	1.85	1.76
O(2) - O(W6) - O(W9)	125·4°	126·3°	130·0°
$O(W_6) - H(62) - O(2)$	142	144	154
$O(W_{6}) - H(6_{9}) - O(W_{9})$	165	168	171
$O(W7) \rightarrow O(3)$	2·768 Å	2·75 Å	2·70 Å
$O(W7) \rightarrow O(4)$	2.828	2.77	2.75
H(73)–O(3)	1.80	1.80	1.73
H(74)-O(4)	1.88	1.80	1.79
O(3) - O(W7) - O(4)	117·1°	119·4°	118·7°
O(W7) - H(73) - O(3)	177	165	176
O(W7)-H(74)-O(4)	167	180	168
$O(W8) \rightarrow O(3)$	2·715 Å	2·73 Å	2.68 A
$O(W8) \rightarrow O(4)$	2.796	2.70	2.72
H(83)–O(3)	1.75	1.77	1.72
H(84)–O(4)	1.87	1.76	1.79
O(3) - O(W8) - O(4)	104·3°	105.5	105.2
O(W8) - H(83) - O(3)	171	169	173
O(W8) - H(84) - O(4)	158	163	167
$O(W9) \rightarrow O(1)$	2.931 A	2.82 A	2.79 A
$O(W9) \rightarrow O(2)$	3.115	3.23	3.00
H(91)-O(1)	2.05	1.82	1.84
H(92) - O(2)	2.16	2.32	2.07
O(1) - O(W9) - O(2)	125.5	123.2	122.1
$O(W_{9}) - H(9_{1}) - O(1)$	150	1/4	161
O(W9) - H(92) - O(2)	100	1.00	2.78
Mean of all IV H bonds	2.031	2.03	2.70
Mean of 8 bonds donated $h_{1} O(W_{5}) O(W_{5})$			
O(W7) O(W8)	2,701	2.70	2.75
O(W 7), O(W 6)	2 / 91	21)	215
by $O(2)$ $A_{\rm p} = -0.17 \text{ yr}$	3.003	3.07	2.90
Mean of 2 honds accented	5 005	507	2,0
by $\Omega(3)$ An $-0.17$ y u	2.742	2.74	2.69
Mean of 6 bonds accented	2 / 72		2 4/
by $O(1)$ $O(4)$ $O(W^{0})$			
$A_n = 0$	2.814	2.79	2.77
$\Delta p = 0$	- 01-T		- · ·

Table 5 (cont.	nt.)	(cor	5	le	Tabl
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coordination have a mean length of 2.716 Å, while the corresponding bonds in the Mg salt have a mean length of 2.742 Å. Water molecules bonded into the square-planar coordination around a Cu atom seem to form stronger hydrogen bonds than water molecules bonded to Mg atoms.

The mean bond length of a hydrogen bond between oxygen atoms can be predicted with a mean deviation of 0.08 Å from the empirical relation (Baur, 1970, 1972):

$$d(O-H.O.O) = (2.85 - 0.38\Delta p) \text{ Å},$$

where  $\Delta p$  is the difference between the values of  $p_0$  received by the donor atom and the acceptor atom of the hydrogen bond. The predicted hydrogen bond

lengths for MgSO<sub>4</sub>.5H<sub>2</sub>O are compared with the observed values in Table 7. The same formalism can be applied to the hydrogen bonds in CuSO<sub>4</sub>.5H<sub>2</sub>O. Since the differences in Cu-O bond length within the distorted coordination octahedra around Cu2+ are not due to  $\Delta p_0$  variations, but are due instead to an inherent asymmetry of the Cu<sup>2+</sup> ions, it appears to be reasonable to assign different bond strengths to the short (0.4v.u.) and the long (0.2v.u.) Cu-O bonds (Table 6). Therefore, the average  $\Delta p$  for the hydrogen bonds in the Cu salt is +0.10v.u., as compared with 0.0v.u. for the Mg salts. The individual and the mean deviations of observed and calculated hydrogen bond lengths (Table 7) are within the range of values found in the much larger sample of 392 hydrogen bonds investigated previously (Baur, 1970).

#### Crystal symmetry of Cu salts

The detailed geometry of the chains of octahedra and tetrahedra is different in the three salts, because of the slightly differing dimensions of the octahedral and tetrahedral groups. For one, these differences are reflected in the unit-cell parameters. Secondly, the chains of octahedra and tetrahedra connected by the bonds -M(1)-O(1)-T-O(2)-M(2) are more kinked in CuSO<sub>4</sub>.5H<sub>2</sub>Oand MgCrO<sub>4</sub>.5H<sub>2</sub>Othan in MgSO<sub>4</sub>.5H<sub>2</sub>O. The centroids of the tetrahedral groups are displaced by about 0.25 Å in the Cu salt, and by about 0.15 Å in the chromate, away from the line M(1)-M(2) as compared with MgSO<sub>4</sub>.5H<sub>2</sub>O. Such changes are possible, because the hydrogen bonds that connect the chains are weak compared with the M-O and T-O bonds and are able to adjust themselves to the changes. It appears that this ability of a crystal structure type to absorb changes in the size and distortion of the MO<sub>6</sub> coordination polyhedron is a necessary condition for the occurrence of strictly isostructural Mg and Cu salts. However, it is clearly not a sufficient condition, otherwise there should be more examples of strictly

isostructural pairs, especially among highly hydrated salts. No additional explanation for the occurrence or absence of isostructural pairs of Cu and Mg salts can be offered here; however, it should be pointed out that the question is too complex to be solved simply by a reference to the static Jahn-Teller effect. Occasionally, the opinion is voiced that the lowering of the symmetry of a Cu compound, as compared with the analogous Mg compound (or other compounds with firstrow transition elements), is due to the distortion of the Cu coordination by the Jahn-Teller effect. For example, monoclinic CuF<sub>2</sub> crystallizes in a distorted rutile type (Billy & Haendler, 1957), while the difluorides of Mg, Mn, Fe, Co, Ni, and Zn (Baur, 1958) represent the undistorted tetragonal rutile type. According to Creighton (1971), this is a most convincing example of a distortion caused by the Jahn-Teller effect, because in the tetragonal rutile compounds there is 'an almost perfect regular octahedral arrangement', while in CuF, the coordination is tetragonal with four short and two long bonds. Actually, however, the local symmetry around the cations is orthorhombic (mmm) even in the tetragonal rutile-type crystals, and this ortho-



Fig. 1. MgSO<sub>4</sub>. 5H<sub>2</sub>O, view parallel to [001]. Polyhedral representation, except for the O(W9) atoms, which are shown as circles. Some of the hydrogen bonds are indicated as arrows pointing from the donor to the acceptor atoms. The complete hydrogen bonding environment of every oxygen atom is displayed once. The heights of some of the central atoms of the polyhedra and of selected oxygen atoms are indicated in fractional coordinates  $\times$  100.

Table 6. MgSO<sub>4</sub>. 5H<sub>2</sub>O and CuSO<sub>4</sub>. 5H<sub>2</sub>O, electrostatic bond strengths (Pauling, 1960, p. 547; Baur, 1970)

The hydrogen atom contribution to the hydrogen bond donor is identified as H(d), to the acceptor as H(a).

/		1	Mg salt					Cu salt	
	Mg(1)	Mg(2)	Š	H(a)	H(d)	po	Cu(1)	Cu(2)	po
<b>O</b> (1)	$\frac{1}{2}$			$\frac{1}{\epsilon}$		2·00 v.u.	$\frac{2}{10}$		1·87 v.u.
O(2)	3	$\frac{1}{2}$	6	$2 \times \frac{3}{4}$		2.17	10	$\frac{2}{10}$	2.04
O(3)		3	4 6	$2 \times \frac{1}{4}$		1.83		10	1.83
O(4)			6	$3 \times \frac{1}{2}$		2.00			2.00
O(W5)	1		4 6	- 6	$2 \times \frac{5}{5}$	2.00	4		2.07
O(W6)	3		4		2 × š	2.00	<u>4</u>		2.07
O(W7)	3	1			2 × §	2.00	10	4	2.07
O(W8)		3 1			2 × 5	2.00		<u>10</u> <u>4</u>	2.07
O(W9)		3		$2 \times \frac{1}{6}$	$2 \times \frac{3}{5}$	2.00		10	2.00

Table 7. Comparison of observed and calculated [equation (1)] hydrogen bond lengths in MgSO<sub>4</sub>.5H<sub>2</sub>O and CuSO<sub>4</sub>.5H<sub>2</sub>O

	MgSO <sub>4</sub> .5H <sub>2</sub> O				CuSO <sub>4</sub> .5H <sub>2</sub> O					
	∆p	dobs	$d_{calc}$	$ \varDelta $	∆p	$d_{\rm obs}$	$d_{calc}$	]⊿		
$O(W5) \rightarrow O(4)$	0·0 v.u.	2∙80 Å	2·85 Å	0∙05 Å	0.07 v.u.	2·83 Å	2·82 Å	0∙01 Å		
$O(W5) \rightarrow O(W9)$	0.0	2.76	2.85	0.09	0.07	2.76	2.82	0.06		
$O(W_6) \rightarrow O(2)$	-0.12	2.89	2.91	0.02	0.03	2.80	2.84	0.04		
$O(W6) \rightarrow O(W9)$	0.0	2.77	2.85	0.08	0.02	2.75	2.82	0.02		
$O(W7) \rightarrow O(3)$	0.17	2.77	2.79	0.02	0.24	2.70	2.76	0.06		
$O(W7) \rightarrow O(4)$	0.0	2.83	2.85	0.02	0.07	2.75	2.82	0.07		
$O(W8) \rightarrow O(3)$	0.17	2.72	2.79	0.07	0.24	2.68	2.76	0.08		
$O(W8) \rightarrow O(4)$	0.0	2.80	2.85	0.05	0.07	2.72	2.82	0.10		
$O(W9) \rightarrow O(1)$	0.0	2.93	2.85	0.08	0.13	2.79	2.80	0.01		
$O(W9) \rightarrow O(2)$	-0.17	3.12	2.91	0.21	-0.04	3.00	<b>2·</b> 87	0.13		
Mean	0.0			0.02	0.10			0.06		

rhombic symmetry is very clearly displayed in all crystals (Baur & Khan, 1971). CuF<sub>2</sub> could crystallize in the undistorted rutile type and have a very distorted coordination octahedron, of the same type as in the monoclinic  $CuF_2$  phase, since the lattice symmetry of a crystal does not have to coincide with a local site symmetry. The difference between  $CuF_2$  and the undistorted rutile type may lie in the possibility of achieving a 2+2+2 coordination (with three different Cu-F bond lengths), or in a nonplanar coordination of Cu around F. Both these arrangements definitely are not allowable by the symmetry of the undistorted rutile type. In the crystal structure of CuF<sub>2</sub> as reported by Billy & Haendler (1957), there is no indication of these additional distortions. However, the structure has not been determined with sufficient accuracy to exclude them. The finding of such distortions could give a clue as to why  $CuF_2$  crystallizes in a distorted rutile type. The presence of the Jahn-Teller effect, by itself, is not a sufficient explanation.

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